Antenna Array Designer Pro v2.0
New Leaf Tools LLC

Parameter	Value
Array Geometry	URA (12x12)
Array Elements	144
Grid	12 rows x 12 cols, dx=0.50λ dy=0.50λ
Element Spacing	0.50 λ
Mainbeam Direction	0°
Null Directions	15.0@45.0, 14.9@-135.0, 30.0@90.1, 30.0@-89.9, 45.0@22.5, 45.0@-157.5, 60.0@0.0, 60.0@180.0
Weight Mode	complex (288 DOF)
SLL Mask Target	-40 dB
Solve ID	NLT-20260427-120844-6ADD70EC
Generated	2026-04-27 12:10:15

✔ CERTIFIED

What CERTIFIED Means

CERTIFIED indicates the solution passed all five minimum quality checks listed below. It is a verified quality floor, not merely the output of an optimizer. Standard tools (MATLAB, Scipy, MVDR) produce a single result with no independent verification and no feedback on whether it meets operational requirements. Our engine generates multiple candidate solutions, independently certifies each one against these checks, ranks them, and selects the best. Solutions that fail any check are flagged, not silently accepted.

Check	Requirement	Result
Residual	< 1e-8	4.65e-13 PASS
Null Depths	≤ –30 dB all nulls	PASS (All nulls <= -30 dB)
Mainbeam Pointing	≤ 1.0 dB gain deficit	PASS (Local peak at (0.00°, 15.00°), error 0.000°. Gain deficit 0.00 dB.)
Visible Region	All nulls |θ| ≤ 90°	PASS
Amplitude Bounds	max(|w_k|) ≤ 1.0	INFO (max=1.00 raw, exported normalized to 1.0)

Executive Results

Metric	Value
Status	CERTIFIED
SLL	-18.0 dB
Directivity	23.2 dBi
Beamwidth	9.4°
Mainbeam Gain	144.000
Pointing Error	0.0000°
Residual	4.65e-13
Health Grade	EXCELLENT
Solve Time	68652 ms

Solve Trace

Metric	Value
Solutions Found	107
Solutions Certified	107

Null Depth Verification

Direction	Depth (dB)	Threshold	Status
15.0, 45.0	-319.5	-30 dB	PASS
14.9, -135.0	-319.4	-30 dB	PASS
30.0, 90.1	-332.6	-30 dB	PASS
30.0, -89.9	-330.6	-30 dB	PASS
45.0, 22.5	-303.3	-30 dB	PASS
45.0, -157.5	-305.7	-30 dB	PASS
60.0, 0.0	-296.2	-30 dB	PASS
60.0, 180.0	-299.2	-30 dB	PASS

Operational Impact

SINR (our weights): **33.7 dB** | SINR (uniform): 22.3 dB | **Improvement: +11.4 dB**

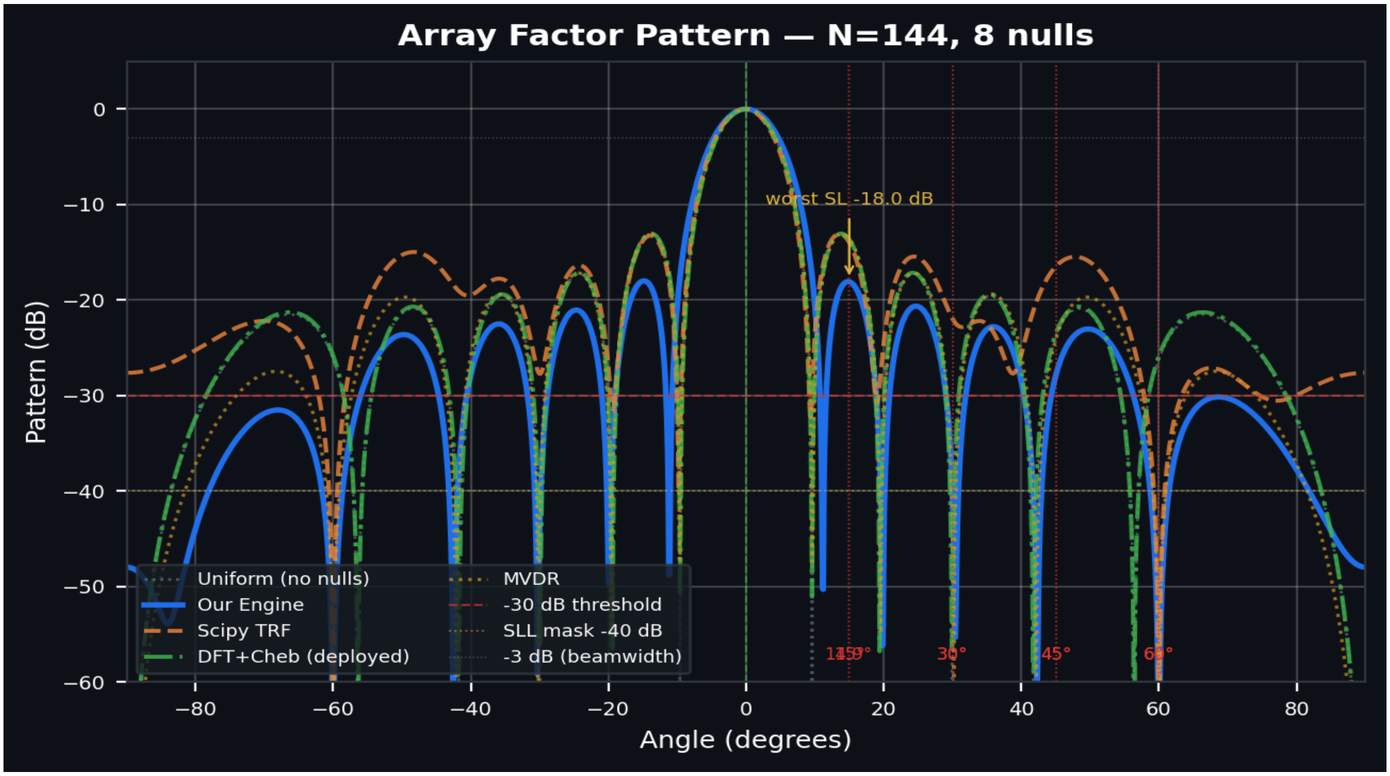
Jammer Direction	Suppression (dB)
15.0@45.0	49
14.9@-135.0	48
30.0@90.1	37
30.0@-89.9	37
45.0@22.5	59
45.0@-157.5	60
60.0	47

Jammer Direction	Suppression (dB)
60.0@180.0	48

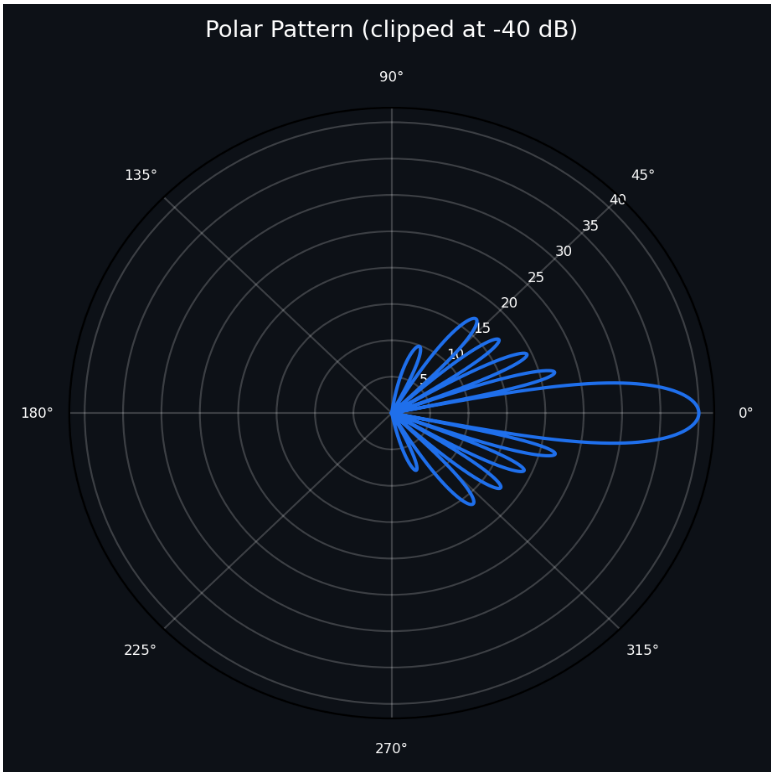
SINR proxy uses ±1° windowed averaging around each jammer direction (41 samples). Assumes equal-power unit interference per null direction. Actual SINR depends on interference environment, jammer bandwidth, and receiver NF.

Array Factor Pattern

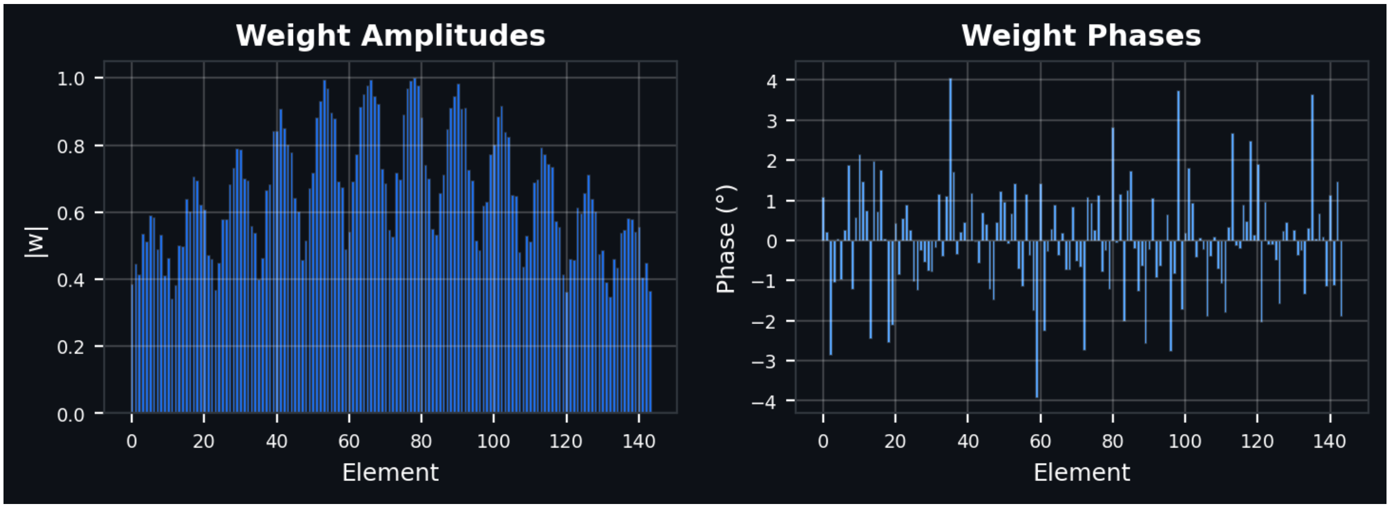
Our Engine (blue) vs baseline methods. Null markers in red, mainbeam in green, SLL mask in amber.



Polar Pattern



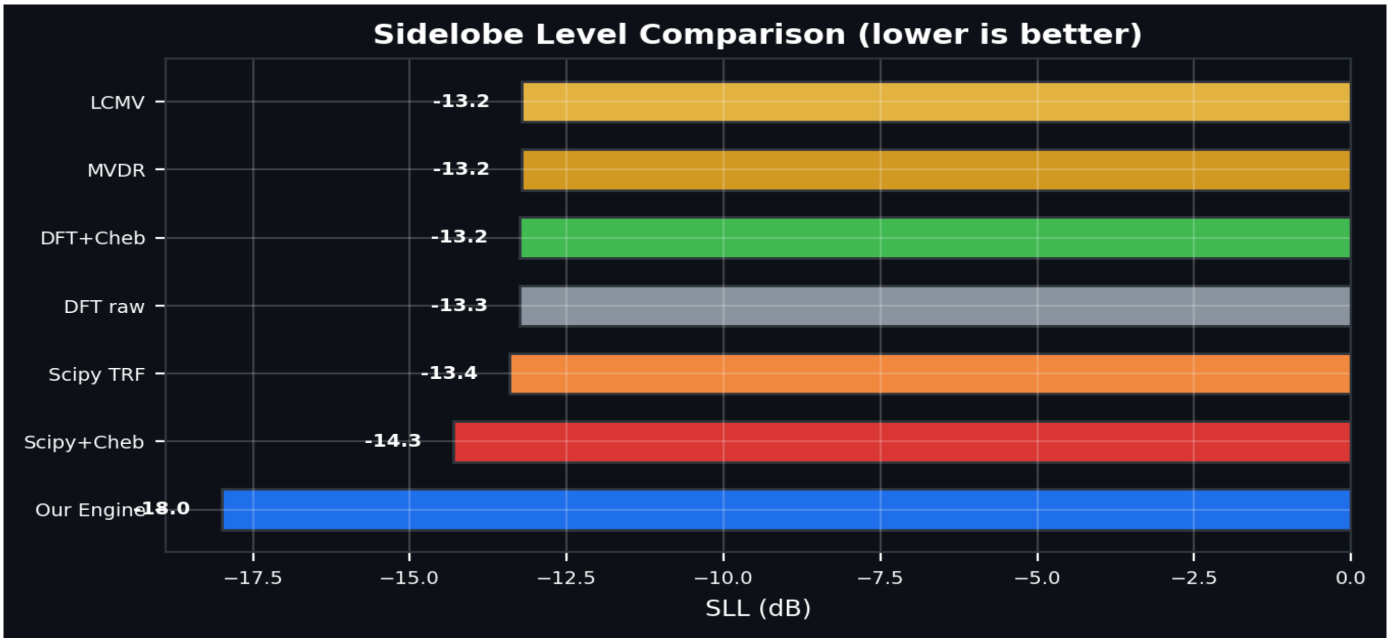
Weight Distribution



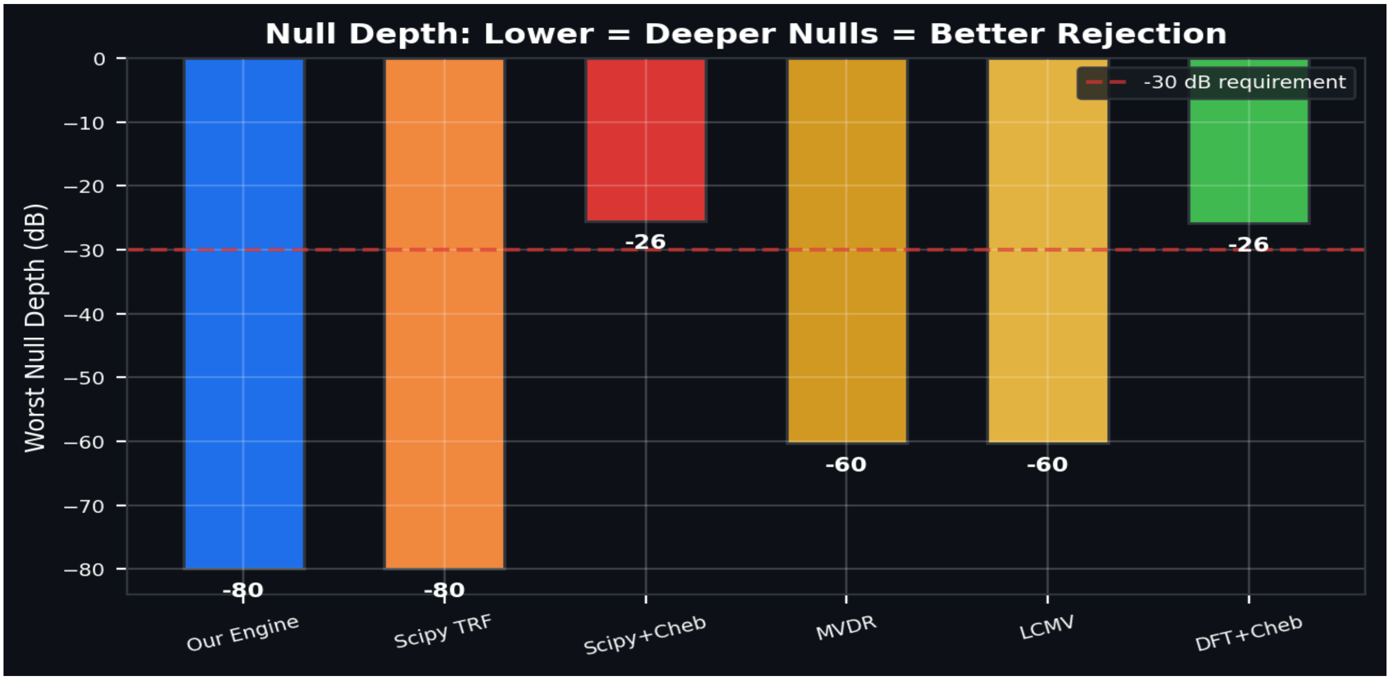
Amplitude and phase per element. These weights are programmed directly into beamforming hardware. Full float64 precision table follows.

Baseline Comparison — 7-Method Table

Method	Worst Null	SLL	Null Control
Our Engine	-296 dB	-18.0 dB	YES (certified)
DFT raw (3GPP)	none	-13.3 dB	NONE
DFT + Chebyshev 50dB	none	-13.2 dB	NONE
Scipy TRF raw	-296 dB	-13.4 dB	YES (uncertified)
Scipy TRF + Cheb 50dB	-26 dB	-14.3 dB	DESTROYED by taper
MVDR/Capon	-60 dB	-13.2 dB	YES (shallow)
LCMV	-60 dB	-13.2 dB	YES (shallow)



Null Depth Comparison — Worst Null Per Method

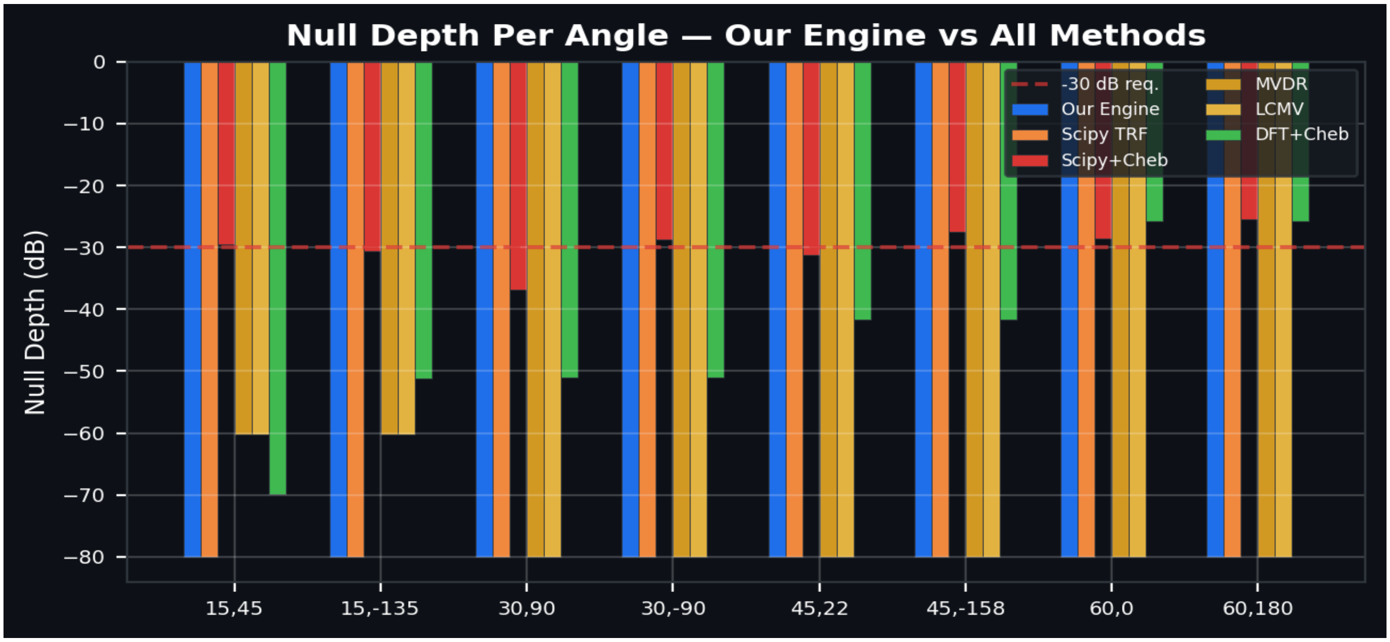


Our actual null depth: -296 dB (clipped to –80 dB on chart for visibility). Scipy TRF achieves -296 dB nulls raw, but Chebyshev tapering destroys them to -26 dB — a 270 dB degradation.

Our engine outperforms the industry-deployed DFT+Chebyshev codebook by 4.7 dB on SLL while providing -296 dB certified null control. Chebyshev tapering destroys Scipy TRF null depths from -296 dB to -26 dB — a 270 dB degradation — and worsens SLL. Our weights are already optimal and require no post-processing. SLL advantage over Scipy TRF raw: 4.6 dB better sidelobes.

Null Depth Per Angle — All Methods

Our engine achieves machine-precision nulls that survive because they require no post-processing. Chebyshev tapering destroys Scipy TRF nulls. DFT codebooks have zero null control.



Note: bars clipped at –80 dB for visibility. Our actual null depth: -296 dB.

Hardware Robustness

Robust Mode: **NOT ENABLED**

Metric	Value
Phase Noise Survival	0% (σ=0°)
Dead Element Survival	0%
Robustness Grade	ROBUST

Enable Robust Mode to test against your specific hardware conditions (phase errors, element failures, quantization) before deployment.

Dead Element Impact Analysis

Each row shows null depth degradation if that element fails. Critical elements are flagged — these need redundancy or monitoring in hardware.

Array has 144 elements — showing summary only. 0 critical element(s) identified out of 144.

No critical elements found — all elements can fail individually without degrading nulls below -20 dB.

Summary: 0 critical / 144 total elements. All elements are resilient to individual failure.

All Solutions Found

The engine found 107 certified solutions and ranked 107 unique solutions. The selected solution (★) has the best combined performance. Null depths below –250 dB are at float64 machine precision and physically equivalent — simulation confirms zero difference after real hardware impairments. When all nulls are equivalent, SLL determines ranking. Standard tools produce a single result with no alternatives.

Rank	Certified	Selected	SLL (dB)	Worst Null (dB)	Dir. (dBi)	Health	Residual
1	YES	★	-18.0	-296	23.2	EXCELLENT	4.65e-13
2	YES		-17.9	-309	23.2	EXCELLENT	1.20e-13
3	YES		-17.9	-300	23.1	EXCELLENT	1.96e-13
4	YES		-17.9	-315	23.1	EXCELLENT	1.26e-13
5	YES		-17.8	-302	23.1	EXCELLENT	1.13e-13
6	YES		-17.8	-308	23.1	EXCELLENT	1.92e-13
7	YES		-17.8	-286	23.1	EXCELLENT	9.23e-13
8	YES		-17.7	-306	23.1	EXCELLENT	1.44e-13
9	YES		-17.6	-295	23.1	EXCELLENT	3.34e-13
10	YES		-17.6	-309	23.2	EXCELLENT	1.30e-13
11	YES		-17.6	-299	23.1	EXCELLENT	4.23e-13
12	YES		-17.6	-300	23.1	EXCELLENT	2.51e-13
13	YES		-17.6	-295	23.0	EXCELLENT	3.69e-13

Rank	Certified	Selected	SLL (dB)	Worst Null (dB)	Dir. (dBi)	Health	Residual
14	YES		-17.5	-294	23.1	EXCELLENT	6.44e-13
15	YES		-17.4	-291	23.1	EXCELLENT	6.41e-13

... and 92 more solutions

Mutual Coupling Matrix

STATUS: DISABLED — No mutual coupling matrix was loaded. All results assume ideal isotropic elements with no inter-element coupling. For production deployments, upload a measured or simulated NxN coupling matrix (from HFSS, CST, or FEKO) to account for real element interactions. The pattern and null depths will be recomputed using the actual coupling coefficients.

Certified Weights

Full float64 precision — copy to hardware.

Elem	Amplitude	Phase (deg)	Re(w)	Im(w)
0	3.8519226067933321e-01	1.086654632651	3.8512298634745173e-01	7.3050033590613128e-03
1	4.4675264905301854e-01	0.211049080533	4.4674961824579490e-01	1.6456102594817955e-03
2	4.1518220747770890e-01	-2.845726131805	4.1467021875239624e-01	-2.0612498293587654e-02
3	5.3669711414658938e-01	-1.044982750415	5.3660785346390005e-01	-9.7879484133701206e-03
4	5.1242536049630638e-01	0.053290751283	5.1242513885075991e-01	4.7660628290866343e-04
5	5.9154016623687211e-01	-0.967504078747	5.9145583193229800e-01	-9.9883504553781276e-03
6	5.8509590677270285e-01	0.264583092402	5.8508966834417486e-01	2.7018732529111070e-03
7	4.9034086808457322e-01	1.890408996347	4.9007400101667731e-01	1.6175303441933835e-02
8	5.3286085700914954e-01	-1.201189132348	5.3274376009769620e-01	-1.1170452519643300e-02
9	4.1111301092417057e-01	0.576070566872	4.1109223149201224e-01	4.1333954631755515e-03
10	4.6239330040875232e-01	2.142320790552	4.6207011281604621e-01	1.7285112237559269e-02
11	3.4296728236826096e-01	1.472703043800	3.4285399437344050e-01	8.8144947244482106e-03
12	3.8308558976353363e-01	0.744953990030	3.8305321002493348e-01	4.9807001584676021e-03
13	5.0033205740205766e-01	-2.442784133423	4.9987739665176012e-01	-2.1325008343135110e-02
14	4.9762107048657495e-01	1.988800184801	4.9732131841155891e-01	1.7269512025348484e-02
15	6.4028542142474776e-01	0.712148497047	6.4023596372814440e-01	7.9581177523886694e-03
16	6.0345726217444451e-01	1.759067800550	6.0317288004487335e-01	1.8524147738806576e-02
17	7.0741124175293779e-01	0.057202082557	7.0741088920324435e-01	7.0625428040484953e-04
18	6.9581048530599032e-01	-2.537877692797	6.9512801135841973e-01	-3.0810376282130177e-02
19	6.2256605553974165e-01	-2.096388023708	6.2214937322709951e-01	-2.2773908391839418e-02
20	6.0780260050667478e-01	0.423699806322	6.0778598162310404e-01	4.4946329234332987e-03
21	4.7206466850099599e-01	-0.844356061198	4.7201340956024385e-01	-6.9564676574190038e-03
22	4.5932517582902738e-01	0.546529478887	4.5930427955376074e-01	4.3813164651431927e-03
23	3.6775292034044627e-01	0.893413691446	3.6770821316036839e-01	5.7341427725270859e-03

Elem	Amplitude	Phase (deg)	Re(w)	Im(w)
24	4.4838737651520250e-01	0.270454827287	4.4838238115527729e-01	2.1165272758904125e-03
25	5.7859359881222761e-01	-1.012654093481	5.7850323192891462e-01	-1.0225616572386067e-02
26	5.7811628305786611e-01	-1.223948364897	5.7798438159293408e-01	-1.2348739663477310e-02
27	6.8274745933436809e-01	-0.245705609481	6.8274118143557916e-01	-2.9278660290678695e-03
28	7.3177135710515318e-01	-0.536766888870	7.3173924502748644e-01	-6.8553895674665268e-03
29	7.8979234821456068e-01	-0.746606052447	7.8972529578345607e-01	-1.0291282621857521e-02
30	7.8613537377756360e-01	-0.772239918878	7.8606397019558327e-01	-1.0595313338765800e-02
31	7.0085644726402563e-01	-0.163944094591	7.0085357816808314e-01	-2.0054028389345636e-03
32	6.9582726209051771e-01	1.154231352559	6.9568607427261608e-01	1.4016587728190021e-02
33	5.5833759736193700e-01	-0.387985174810	5.5832479617570452e-01	-3.7808204478115689e-03
34	5.3845342897909654e-01	1.110115801836	5.3835236512186835e-01	1.0431977139423978e-02
35	4.0008570567264595e-01	4.071375233213	3.9907604071129393e-01	2.8405732058455060e-02
36	4.6360610862069862e-01	1.722557299068	4.6339660644790420e-01	1.3935891898017652e-02
37	6.6624607451616569e-01	-0.335198575848	6.6623467300298644e-01	-3.8977296986722801e-03
38	6.8437416930236761e-01	0.217720879475	6.8436922826673052e-01	2.6005787635213738e-03
39	8.4092866392058851e-01	0.452057631986	8.4090248997144823e-01	6.6347692562898301e-03
Elem	Amplitude	Phase (deg)	Re(w)	Im(w)
40	8.4338067793896632e-01	0.025435016449	8.4338059483678474e-01	3.7439757158300792e-04
41	9.0810599642830514e-01	1.185983323343	9.0791145910282078e-01	1.8795828761512517e-02
42	8.4980299922886848e-01	-0.037646833232	8.4980281578640071e-01	-5.5837253217829916e-04
43	8.0082302231356839e-01	-0.559303877960	8.0078486713400032e-01	-7.8172652903418897e-03
44	7.7816955708033286e-01	0.709529881922	7.7810988999725361e-01	9.6363195808018710e-03
45	6.4365717702562808e-01	0.401946607119	6.4364133851615390e-01	4.5154058216842746e-03
46	6.0139570178164115e-01	-1.207204422876	6.0126221734630381e-01	-1.2670284655001448e-02
47	4.5734577221219797e-01	-1.472499056490	4.5719474487870687e-01	-1.1752472746865736e-02
48	5.1488803008636364e-01	0.451619707353	5.1487203520377023e-01	4.0584345927441945e-03

Elem	Amplitude	Phase (deg)	Re(w)	Im(w)
49	6.7166461787037424e-01	1.229724770925	6.7150992283302580e-01	1.4414660444822643e-02
50	7.1831491163064765e-01	0.959440414595	7.1821420327846330e-01	1.2027904224234465e-02
51	8.8256924967641737e-01	-0.078073359911	8.8256843030803100e-01	-1.2026213088981283e-03
52	9.3230366217988869e-01	0.674387249414	9.3223908251057752e-01	1.0973219849643214e-02
53	9.9595819806495534e-01	1.421576885073	9.9565166009466377e-01	2.4708380026484303e-02
54	9.7015295474009178e-01	-0.713287814126	9.7007777705469733e-01	-1.2077336442584021e-02
55	8.9789660751571210e-01	-1.144971217115	8.9771733002862664e-01	-1.7941938426452950e-02
56	8.7919988943690941e-01	1.168434126173	8.7901707698274378e-01	1.7928300493540307e-02
57	6.9246451898810668e-01	-0.374469026402	6.9244972952111916e-01	-4.5257202254429833e-03
58	6.7497065776267895e-01	-1.739334683026	6.7465967050490805e-01	-2.0487016249135140e-02
59	4.8929573114945935e-01	-3.917207042236	4.8815264214357301e-01	-3.3426194658272491e-02
60	5.4023827388414825e-01	1.412850214002	5.4007403335879944e-01	1.3320325104231109e-02
61	6.9064311713770299e-01	-2.244776992902	6.9011312518015577e-01	-2.7051611851450788e-02
62	7.7254005784057467e-01	-0.275217070391	7.7253114542051537e-01	-3.7108386632975724e-03
63	9.1287822533295804e-01	0.285585071911	9.1286688547295425e-01	4.5501312024025953e-03
64	9.5217221756264803e-01	0.892046470400	9.5205681739741843e-01	1.4823911267584831e-02
65	9.7825223547295714e-01	-0.357405330300	9.7823320295576610e-01	-6.1021998280292328e-03
66	9.9477525180951099e-01	0.188296344417	9.9476987984545229e-01	3.2692147283834650e-03
67	9.4540008877749104e-01	-0.716509499255	9.4532616594089136e-01	-1.1822345282654129e-02
68	9.2363338339927858e-01	-0.736334598861	9.2355711075575930e-01	-1.1869713647463521e-02
69	7.3026209811500442e-01	0.834870693995	7.3018457445843243e-01	1.0640449534037861e-02
70	6.8676619208350143e-01	-0.513510455867	6.8673860985690560e-01	-6.1550240192412920e-03
71	5.4721551658907508e-01	-0.655639093508	5.4717968978354437e-01	-6.2616838176677743e-03
72	5.2620922336994569e-01	-2.728340215896	5.2561274022130289e-01	-2.5047835767878904e-02
73	7.1660613772359616e-01	1.095691437941	7.1647510838052553e-01	1.3703127170250013e-02
74	6.9623938259998075e-01	0.944674491880	6.9614475063372616e-01	1.1478852220914637e-02

Elem	Amplitude	Phase (deg)	Re(w)	Im(w)
75	8.9135571025063365e-01	0.260284466821	8.9134651271358190e-01	4.0492554559827020e-03
76	9.6852908803938431e-01	1.145269724933	9.6833560695689957e-01	1.9358374875348257e-02
77	9.9305950742130322e-01	-0.783049423429	9.9296676645822612e-01	-1.3571513892564483e-02
78	9.9999999999999978e-01	-0.238644385060	9.9999132585749573e-01	-4.1651182177013073e-03
79	9.7823002083523003e-01	-1.210041844978	9.7801187327391914e-01	-2.0657913702352450e-02

Elem	Amplitude	Phase (deg)	Re(w)	Im(w)
80	8.8375488456336038e-01	2.824880011329	8.8268097279541857e-01	4.3554520483320522e-02
81	7.4105011259891529e-01	-0.042490175902	7.4104990882464095e-01	-5.4955787351348821e-04
82	6.9915760479097055e-01	1.164052158914	6.9901331693893520e-01	1.4203488271364939e-02
83	5.4928758469827421e-01	-2.019926599060	5.4894627335867852e-01	-1.9360776567140640e-02
84	5.3270456105943620e-01	1.259670297517	5.3257582285741067e-01	1.1710776288462947e-02
85	6.5651171639226069e-01	1.731199539316	6.5621205662686688e-01	1.9833569972393043e-02
86	7.1301927261138542e-01	-0.184414633597	7.1301557929601456e-01	-2.2949502038325325e-03
87	8.4826984024090557e-01	-1.259109433632	8.4806502218313928e-01	-1.8639742805171979e-02
88	9.1059795358825946e-01	-0.634631452874	9.1054209497446537e-01	-1.0085948574017302e-02
89	9.4480883132762250e-01	-2.561411667524	9.4386486800642944e-01	-4.2223674613579169e-02
90	9.8250249994461936e-01	-0.229013745262	9.8249465154832905e-01	-3.9270951547039704e-03
91	9.0779857676562259e-01	1.067047301113	9.0764115345985086e-01	1.6905399248802674e-02
92	9.1051559301441798e-01	-0.920007421526	9.1039821535169618e-01	-1.4619665072902403e-02
93	7.2599404027498815e-01	-0.623090586920	7.2595111076852303e-01	-7.8950167035045666e-03
94	6.9364024908130717e-01	-0.001589537335	6.9364024881437536e-01	-1.9243425638947433e-05
95	5.1623371892621228e-01	0.656467710304	5.1619983504932410e-01	5.9146302875284348e-03
96	4.8682266983596750e-01	-2.758619202911	4.8625851911727891e-01	-2.3429990441532784e-02
97	6.2012187512576578e-01	-0.826075693762	6.2005742348830373e-01	-8.9404466634593286e-03
98	6.2980458194921118e-01	3.747898777932	6.2845763218651818e-01	4.1168142910940821e-02
99	7.7298130277264665e-01	-1.716490960987	7.7263445070471271e-01	-2.3153833814833796e-02

Elem	Amplitude	Phase (deg)	Re(w)	Im(w)
100	8.0147748950647235e-01	0.192396054567	8.0147297086395763e-01	2.6913119661845443e-03
101	8.8459035454019741e-01	1.816624447629	8.8414576333670736e-01	2.8042191770671979e-02
102	9.1775540857308180e-01	0.945143204585	9.1763054462110782e-01	1.5138478903558671e-02
103	8.3874519799426694e-01	-0.403539842652	8.3872439500085016e-01	-5.9073165565801328e-03
104	8.2606610855610285e-01	0.075890977338	8.2606538392032625e-01	1.0941634187689165e-03
105	6.5097005962939658e-01	-0.226281018810	6.5096498293009719e-01	-2.5709011490698042e-03
106	6.4731032986719172e-01	-1.879101613800	6.4696223398952912e-01	-2.1225714217673004e-02
107	4.7980697601291328e-01	-0.381260208489	4.7979635336818044e-01	-3.1927300625757883e-03
108	4.3773535822033760e-01	0.105162023280	4.3773462090356691e-01	8.0342933592671953e-04
109	5.2861265024919146e-01	-0.692272477458	5.2857406590351608e-01	-6.3867720876396254e-03
110	5.1331317323225867e-01	-1.061625055938	5.1322506082953878e-01	-9.5105599355151827e-03
111	6.9012066150574414e-01	-1.797250408713	6.8978116810159462e-01	-2.1644111659423790e-02
112	6.9725400125040982e-01	0.332280669809	6.9724227593073573e-01	4.0436264211908676e-03
113	7.9325454364565506e-01	2.672984694370	7.9239146279244776e-01	3.6993792832328579e-02
114	7.7173788835249824e-01	-0.132853485280	7.7173581372432298e-01	-1.7894507621244574e-03
115	7.4374814514158516e-01	-0.204677281705	7.4374339955988811e-01	-2.6568802474280962e-03
116	7.3506645158628958e-01	0.886161342793	7.3497853551231551e-01	1.1368402870728810e-02
117	5.7356451024798283e-01	0.489208205115	5.7354360324229947e-01	4.8972028596055411e-03
118	5.5703669657997357e-01	2.498704168600	5.5650707045897962e-01	2.4285013195256226e-02
119	4.1429498616010346e-01	0.151461138357	4.1429353860002560e-01	1.0951856780636209e-03

Elem	Amplitude	Phase (deg)	Re(w)	Im(w)
120	3.6360794248465333e-01	1.906201251031	3.6340672971353200e-01	1.2094818594707409e-02
121	4.6123704799291104e-01	-2.025288234875	4.6094892535483667e-01	-1.6300388198939354e-02
122	4.5726953108279117e-01	0.957877008394	4.5720563026932093e-01	7.6443251310162556e-03
123	6.1260121039234128e-01	-0.100142144586	6.1260027469314793e-01	-1.0707100640206155e-03
124	5.9754090852754094e-01	-0.096022034027	5.9754006938819748e-01	-1.0014187271787671e-03

Elem	Amplitude	Phase (deg)	Re(w)	Im(w)
125	6.5698451325806162e-01	-0.487297123616	6.5696075224518402e-01	-5.5875460064357049e-03
126	7.1341757312411536e-01	-1.572279172795	7.1314897646150521e-01	-1.9574754512127968e-02
127	6.3861821406258934e-01	0.246809734443	6.3861228904098344e-01	2.7509304474368137e-03
128	6.0201734130419526e-01	0.463794209898	6.0199761789365691e-01	4.8731182352356973e-03
129	4.7392177316629003e-01	-0.005586710828	4.7392177091338039e-01	-4.6210452496140133e-05
130	4.8568098284229777e-01	0.269126213053	4.8567562503560407e-01	2.2813024647328857e-03
131	3.9197474425708917e-01	-0.368830629733	3.9196662277510214e-01	-2.5232450715773069e-03
132	3.4763644242418285e-01	-0.234194877822	3.4763353837333599e-01	-1.4209501667893944e-03
133	4.5932577955655446e-01	-1.325539158731	4.5920286273719529e-01	-1.0625564417997339e-02
134	4.3467867270649080e-01	0.305680421699	4.3467248645901985e-01	2.3190561463143611e-03
135	5.3793409638285405e-01	3.643210864917	5.3684697986069740e-01	3.4182045955232186e-02
136	5.4849731209035868e-01	0.039812820772	5.4849717967293832e-01	3.8113144822111484e-04
137	5.8070116940419414e-01	0.682476073650	5.8065997415198789e-01	6.9168320213326018e-03
138	5.7868638592940358e-01	0.100123432111	5.7868550236268090e-01	1.0112444246173817e-03
139	5.4282850827591478e-01	-1.139193550841	5.4272121608453816e-01	-1.0792173496329089e-02
140	5.5631704062045395e-01	1.144640267465	5.5620602842333344e-01	1.1113218716543815e-02
141	4.0640607803374135e-01	-1.112132312216	4.0632952115539034e-01	-7.8879972362225891e-03
142	4.4792629824320290e-01	1.464181333016	4.4778004773047669e-01	1.1445414468283652e-02
143	3.6655981719335551e-01	-1.880397768935	3.6636242495695592e-01	-1.2028015650349323e-02

Recommendations

1. Program weights into beamforming hardware: 144 elements, d = 0.50λ, mainbeam 0°.
2. For phase-shifter implementation, select appropriate quantization bits and re-solve to see quantization impact.
3. Use the MATLAB verification script to independently confirm all null depths and pattern characteristics before deployment.
4. Solution is ROBUST — suitable for production deployment.

Engineering Notes

SLL vs Null Tradeoff: The SLL mask target (-40 dB) serves as an optimization floor. The achieved SLL (-18.0 dB) reflects the joint SLL + null optimum for this configuration. Standard methods force a choice: apply Chebyshev tapering for better SLL and null control collapses, or optimize for nulls and sidelobes blow out. This engine solves for both simultaneously. The selected solution is the best-ranked candidate from 107 certified solutions, not a single-shot result.

Null Depth and Hardware Headroom: The reported null depths (-296 dB) represent the mathematical solution at float64 precision. In real hardware, quantization (phase shifter bits), mutual coupling, calibration errors, and thermal noise will reduce effective null depth. This is expected and is precisely why the mathematical solution provides 266 dB of headroom above the –30 dB requirement. Standard methods (MVDR, LCMV) start at –18 to –38 dB with 0–12 dB of headroom — leaving no margin for hardware degradation. Use Robust Mode (Monte Carlo phase perturbation, dead element analysis, quantization impact) to verify that headroom survives your specific hardware conditions. The Hardware Robustness section of this report quantifies this.

Verification

All values independently verifiable using the standard AF equation:

```
AF(θ) = Σ w_k · exp(j·k·d_norm·sin(θ)), k=0,...,N-1
d_norm = 2π·d/λ
```

Conventions:

Time convention: exp(+jωt) (IEEE / physics standard)

θ = 0: Broadside (perpendicular to array axis)

Element 0: Phase reference

Normalization: max(|w_k|) = 1.0

Pattern dB: $10 \cdot \log_{10}(|AF(\theta)|^2 / |AF(\theta_{main})|^2)$

If your tool uses exp(-jωt), conjugate the weights.

MATLAB Verification Script

Recommended: Use the standalone .m file download from the Export tab. The .m file is ready to run directly in MATLAB or Octave with no editing required. The script independently computes SLL, directivity, beamwidth, null depths, and pointing error using only the standard AF equation and exported weights — nothing is hardcoded. It prints a PASS/FAIL verification table comparing MATLAB-computed values against our claimed values. The script below is included for reference only — copying from PDF may introduce page headers/footers that must be removed manually.

```
% ================================================================
% INDEPENDENT VERIFICATION SCRIPT
% Antenna Array Designer Pro v2.0 -- New Leaf Tools LLC
% ================================================================
%
% PURPOSE: Independently verify every claimed metric using only the
% standard array factor equation and the exported weights.
% Nothing is hardcoded -- MATLAB computes SLL, directivity,
% beamwidth, null depths, and pointing error from scratch.
%
% EQUATION (universal -- works for ULA, URA, UCA, conformal):
% AF(theta,phi) = sum_n w_n * exp(j*2*pi*r_n . u(theta,phi))
% where r_n = [x_n, y_n, z_n] in wavelengths
% and u = [sin(theta)*cos(phi), sin(theta)*sin(phi), cos(theta)]
%
% CONVENTIONS:
% Time convention: exp(+jwt) -- IEEE / physics standard
% theta = 0: Broadside (perpendicular to array axis/plane)
% Element 0: Phase reference
% Normalization: max(|w_k|) = 1.0
% Pattern dB: 10*log10(|AF|^2 / |AF_main|^2)
% If your tool uses exp(-jwt), conjugate the weights.
%
% GEOMETRY: Uniform Rectangular Array (12x12)
% Elements: 144
% Mainbeam: theta=0 deg, phi=0 deg
% Nulls: theta=[15.0, 14.9, 30.0, 30.0, 45.0, 45.0, 60.0, 60.0] deg, phi=[45.0, -135.0, 90.1, -89.9, 22.5, -157.5, 0.0,
180.0] deg
% ================================================================

clear; close all; clc;

% -- CLAIMED VALUES (from our report -- MATLAB will verify each one) --
claimed_sll_dB = -18.0;
claimed_dir_dBi = 23.2;
claimed_bw_deg = 9.4;
claimed_worst_null = -296.2;
claimed_pointing_err = 0.0000;

% -- Array parameters --
N = 144;

% Element positions (wavelengths) -- URA
positions = [
0.0000000000000000e+00, 0.0000000000000000e+00, 0.0000000000000000e+00;
5.0000000000000000e-01, 0.0000000000000000e+00, 0.0000000000000000e+00;
1.0000000000000000e+00, 0.0000000000000000e+00, 0.0000000000000000e+00;
1.5000000000000000e+00, 0.0000000000000000e+00, 0.0000000000000000e+00;
2.0000000000000000e+00, 0.0000000000000000e+00, 0.0000000000000000e+00;
2.5000000000000000e+00, 0.0000000000000000e+00, 0.0000000000000000e+00;
3.0000000000000000e+00, 0.0000000000000000e+00, 0.0000000000000000e+00;
3.5000000000000000e+00, 0.0000000000000000e+00, 0.0000000000000000e+00;
4.0000000000000000e+00, 0.0000000000000000e+00, 0.0000000000000000e+00;
4.5000000000000000e+00, 0.0000000000000000e+00, 0.0000000000000000e+00;
5.0000000000000000e+00, 0.0000000000000000e+00, 0.0000000000000000e+00;
5.5000000000000000e+00, 0.0000000000000000e+00, 0.0000000000000000e+00;
0.0000000000000000e+00, 5.0000000000000000e-01, 0.0000000000000000e+00;
5.0000000000000000e-01, 5.0000000000000000e-01, 0.0000000000000000e+00;
1.0000000000000000e+00, 5.0000000000000000e-01, 0.0000000000000000e+00;
1.5000000000000000e+00, 5.0000000000000000e-01, 0.0000000000000000e+00;
2.0000000000000000e+00, 5.0000000000000000e-01, 0.0000000000000000e+00;
2.5000000000000000e+00, 5.0000000000000000e-01, 0.0000000000000000e+00;
3.00000000
00000000e+00, 5.0000000000000000e-01, 0.0000000000000000e+00;
3.5000000000000000e+00, 5.0000000000000000e-01, 0.0000000000000000e+00;
4.0000000000000000e+00, 5.0000000000000000e-01, 0.0000000000000000e+00;
4.5000000000000000e+00, 5.0000000000000000e-01, 0.0000000000000000e+00;
5.0000000000000000e+00, 5.0000000000000000e-01, 0.0000000000000000e+00;
```

```
5.5000000000000000e+00, 5.0000000000000000e-01, 0.0000000000000000e+00;
0.0000000000000000e+00, 1.0000000000000000e+00, 0.0000000000000000e+00;
5.0000000000000000e-01, 1.0000000000000000e+00, 0.0000000000000000e+00;
1.0000000000000000e+00, 1.0000000000000000e+00, 0.0000000000000000e+00;
1.5000000000000000e+00, 1.0000000000000000e+00, 0.0000000000000000e+00;
2.0000000000000000e+00, 1.0000000000000000e+00, 0.0000000000000000e+00;
2.5000000000000000e+00, 1.0000000000000000e+00, 0.0000000000000000e+00;
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3.5000000000000000e+00, 1.0000000000000000e+00, 0.0000000000000000e+00;
4.0000000000000000e+00, 1.0000000000000000e+00, 0.0000000000000000e+00;
4.5000000000000000e+00, 1.0000000000000000e+00, 0.0000000000000000e+00;
5.0000000000000000e+00, 1.0000000000000000e+00, 0.0000000000000000e+00;
5.5000000000000000e+00, 1.0000000000000000e+00, 0.0000000000000000e+00;
0.0000000000000000e+00, 1.5000000000000000e+00, 0.0000000000000000e+00;
5.0000000000000000e-01, 1.5000000000000000e+00, 0.0000000000000000e+00;
1.0000000000000000e+00, 1.5000000000000000e+00, 0.0000000000000000e+00;
1.5000000000000000e+00, 1.5000000000000000e+00, 0.0000000000000000e+00;
2.0000000000000000e+00, 1.5000000000000000e+00, 0.0000000000000000e+00;
2.5000000000000000e+00, 1.5000000000000000e+00, 0.0000000000000000e+00;
3.0000000000000000e+00, 1.5000000000000000e+00, 0.0000000000000000e+00;
3.5000000000000000e+00, 1.5000000000000000e+00, 0.0000000000000000e+00;
4.0000000000000000e+00, 1.5000000000000000e+00, 0.0000000000000000e+00;
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5.5000000000000000e+00, 1.5000000000000000e+00, 0.0000000000000000e+00;
0.0000000000000000e+00, 2.0000000000000000e+00, 0.0000000000000000e+00;
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1.0000000000000000e+00, 2.0000000000000000e+00, 0.0000000000000000e+00;
1.5000000000000000e+00, 2.0000000000000000e+00, 0.0000000000000000e+00;
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3.5000000000000000e+00, 2.0000000000000000e+00, 0.0000000000000000e+00;
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3.5000000000000000e+00, 2.5000000000000000e+00, 0.0000000000000000e+00;
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5.0000000000000000e+00, 2.5000000000000000e+00, 0.0000000000000000e+00;
5.5000000000000000e+00, 2.5000000000000000e+00, 0.0000000000000000e+00;
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4.5000000000000000e+00, 3.0000000000000000e+00, 0.0000000000000000e+00;
5.0000000000000000e+00, 3.0000000000000000e+00, 0.0000000000000000e+00;
5.5000000000000000e+00, 3.0000000000000000e+00, 0.0000000000000000e+00;
0.0000000000000000e+00, 3.5000000000000000e+00, 0.0000000000000000e+00;
5.0000000000000000e-01, 3.5000000000000000e+00, 0.0000000000000000e+00;
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2.5000000000000000e+00, 3.5000000000000000e+00, 0.0000000000000000e+00;
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5.5000000000000000e+00, 3.5000000000000000e+00, 0.0000000000000000e+00;
0.0000000000000000e+00, 4.0000000000000000e+00, 0.0000000000000000e+00;
5.0000000000000000e-01, 4.0000000000000000e+00, 0.0000000000000000e+00;
```

```
1.0000000000000000e+00, 4.0000000000000000e+00, 0.0000000000000000e+00;
1.50000000000000
00e+00, 4.0000000000000000e+00, 0.0000000000000000e+00;
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2.5000000000000000e+00, 4.0000000000000000e+00, 0.0000000000000000e+00;
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1.5000000000000000e+00, 4.5000000000000000e+00, 0.0000000000000000e+00;
2.0000000000000000e+00, 4.5000000000000000e+00, 0.0000000000000000e+00;
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3.5000000000000000e+00, 4.5000000000000000e+00, 0.0000000000000000e+00;
4.0000000000000000e+00, 4.5000000000000000e+00, 0.0000000000000000e+00;
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5.0000000000000000e+00, 4.5000000000000000e+00, 0.0000000000000000e+00;
5.5000000000000000e+00, 4.5000000000000000e+00, 0.0000000000000000e+00;
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1.5000000000000000e+00, 5.0000000000000000e+00, 0.0000000000000000e+00;
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3.5000000000000000e+00, 5.0000000000000000e+00, 0.0000000000000000e+00;
4.0000000000000000e+00, 5.0000000000000000e+00, 0.0000000000000000e+00;
4.5000000000000000e+00, 5.0000000000000000e+00, 0.0000000000000000e+00;
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5.0000000000000000e-01, 5.5000000000000000e+00, 0.0000000000000000e+00;
1.0000000000000000e+00, 5.5000000000000000e+00, 0.0000000000000000e+00;
1.5000000000000000e+00, 5.5000000000000000e+00, 0.0000000000000000e+00;
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3.5000000000000000e+00, 5.5000000000000000e+00, 0.000000
0000000000e+00;
4.0000000000000000e+00, 5.5000000000000000e+00, 0.0000000000000000e+00;
4.5000000000000000e+00, 5.5000000000000000e+00, 0.0000000000000000e+00;
5.0000000000000000e+00, 5.5000000000000000e+00, 0.0000000000000000e+00;
5.5000000000000000e+00, 5.5000000000000000e+00, 0.0000000000000000e+00
];

% Optimized weights (display-normalized, max|w|=1)
w_real = [0.38512298634745173, 0.4467496182457949, 0.41467021875239624, 0.5366078534639, 0.5124251388507599,
0.591455831932298, 0.5850896683441749, 0.4900740010166773, 0.5327437600976962, 0.41109223149201224, 0.4620701128160462,
0.3428539943734405, 0.3830532100249335, 0.4998773966517601, 0.4973213184115589, 0.6402359637281444, 0.6031728800448733,
0.7074108892032444, 0.6951280113584197, 0.6221493732270995, 0.607785981623104, 0.47201340956024385, 0.45930427955376074,
0.3677082131603684, 0.4483823811552773, 0.5785032319289146, 0.5779843815929341, 0.6827411814355792, 0.7317392450274864,
0.7897252957834561, 0.7860639701955833, 0.7008535781680831, 0.6956860742726161, 0.5583247961757045, 0.5383523651218683,
0.39907604071129393, 0.4633966064479042, 0.6662346730029864, 0.6843692282667305, 0.8409024899714482, 0.8433805948367847,
0.9079114591028208, 0.8498028157864007, 0.8007848671340003, 0.7781098899972536, 0.6436413385161539, 0.6012622173463038,
0.45719474487870687, 0.5148720352037702, 0.6715099228330258, 0.7182142032784633, 0.882568430308031, 0.9322390825105775,
0.9956516600946638, 0.9700777770546973, 0.8977173300286266, 0.8790170769827438, 0.6924497295211192, 0.674659670504908,
0.488152642143573, 0.5400740333587994, 0.6901131251801558, 0.7725311454205154, 0.9128668854729542, 0.9520568173974184,
0.9782332029557661, 0.9947698798454523, 0.9453261659408914, 0.9235571107557593, 0.7301845744584324, 0.6867386098569056,
0.5471796897835444, 0.5256127402213029, 0.7164751083805255, 0.6961447506337262, 0.8913465127135819, 0.9683356069568996,
0.9929667664582261, 0.9999913258574957, 0.9780118732739191, 0.8826809727954186, 0.741049908824641, 0.6990133169389352,
0.5489462733586785, 0.5325758228574107, 0.6562120566268669, 0.7130155792960146, 0.8480650221831393, 0.9105420949744654,
0.9438648680064294, 0.982494651548329, 0.9076411534598509, 0.9103982153516962, 0.725951110768523, 0.6936402488143754,
0.5161998350493241, 0.4862585191172789, 0.6200574234883037, 0.6284576321865182, 0.7726344507047127, 0.8014729708639576,
0.8841457633367074, 0.9176305446211078, 0.8387243950008502, 0.8260653839203262, 0.6509649829300972, 0.6469622339895291,
0.47979635336818044, 0.4377346209035669, 0.5285740659035161, 0.5132250608295388, 0.6897811681015946, 0.6972422759307357,
0.7923914627924478, 0.771735813724323, 0.7437433995598881, 0.7349785355123155, 0.5735436032422995, 0.5565070704589796,
0.4142935386000256, 0.363406729713532, 0.46094892535483667, 0.45720563026932093, 0.6126002746931479, 0.5975400693881975,
0.656960752245184, 0.7131489764615052, 0.6386122890409834, 0.6019976178936569, 0.4739217709133804, 0.4856756250356041,
0.391966
62277510214, 0.347633538373336, 0.4592028627371953, 0.43467248645901985, 0.5368469798606974, 0.548497179672938З,
0.5806599741519879, 0.5786855023626809, 0.5427212160845382, 0.5562060284233334, 0.40632952115539034, 0.4477800477304767,
```

```
0.3663624249569559];
w_imag = [0.007305003359061313, 0.0016456102594817955, -0.020612498293587654, -0.00978794841337012, 0.0004766062829086634,
-0.009988350455378128, 0.002701873252911107, 0.016175303441933835, -0.0111704525196433, 0.0041333954631755515,
0.01728511223755927, 0.00881449472444821, 0.004980700158467602, -0.02132500834313511, 0.017269512025348484,
0.00795811775238867, 0.018524147738806576, 0.0007062542804048495, -0.030810376282130177, -0.022773908391839418,
0.004494632923433299, -0.006956467657419004, 0.004381316465143193, 0.005734142772527086, 0.0021165272758904125,
-0.010225616572386067, -0.01234873966347731, -0.0029278660290678695, -0.006855389567466527, -0.010291282621857521,
-0.0105953133387658, -0.0020054028389345636, 0.014016587728190021, -0.003780820447811569, 0.010431977139423978,
0.02840573205845506, 0.013935891898017652, -0.00389772969867228, 0.002600578763521374, 0.00663476925628983,
0.0003743975715830079, 0.018795828761512517, -0.0005583725321782992, -0.00781726529034189, 0.009636319580801871,
0.004515405821684274 6, -0.012670284655001448, -0.011752472746865736, 0.0040584345927441945, 0.014414660444822643,
0.01202790422423446 5, -0.0012026213088981283, 0.010973219849643214, 0.024708380026484303, -0.01207733644258402,
-0.017941938426452 95, 0.017928300493540307, -0.004525720225442983, -0.02048701624913514, -0.03342619465827249,
0.013320325104231109, -0.027051611851450788, -0.0037108386632975724, 0.004550131202402595, 0.014823911267584831,
-0.006102199828029233, 0.003269214728383465, -0.011822345282654129, -0.011869713647463 52, 0.01064044953403786,
-0.006155024019241292, -0.006261683817667774, -0.025047835767878904, 0.013703127170250013, 0.011478852220914637,
0.004049255455982702, 0.019358374875348257, -0.013571513892564483, -0.004165118217701307, -0.02065791370235245,
0.04355452048332052, -0.0005495578735134882, 0.01420348827136494, -0.01936077656714064, 0.011710776288462947,
0.019833569972393043, -0.0022949502038325325, -0.01863974280517198, -0.010085948574017302, -0.04222367461357917,
-0.00392709515470397, 0.016905399248802674, -0.014619665072902403, -0.007895016703504567, -1.9243425638947433e-05,
0.005914630287528435, -0.023429990441532784, -0.008940446663459329, 0.04116814291094082, -0.023153833814833796,
0.002691311966184544 3, 0.02804219177067198, 0.015138478903558671, -0.005907316556580133, 0.0010941634187689165,
-0.0025709011490698042, -0.021225714217673004, -0.0031927300625757883, 0.0008034293359267195, -0.006386772087639625,
-0.009510559935515183, -0.02164411165942379, 0.004043626421190868, 0.03699379283232858, -0.0017894507621244574,
-0.002656880247428096, 0.01136840287072881, 0.004897202859605541, 0.024285013195256226, 0.001095185678063621,
0.012094818594707409, -0.016300388198939354,
0.0076443251310162556, -0.0010707100640206155, -0.001001418727178767, -0.005587546006435705, -0.01957475451212797,
0.0027509304474368137, 0.004873118235235697, -4.621045249614013e-05, 0.0022813024647328857, -0.002523245071577307,
-0.0014209501667893944, -0.010625564417997339, 0.002319056146314361, 0.034182045955232186, 0.00038113144822111484,
0.006916832021332602, 0.0010112444246173817, -0.01079217349632909, 0.011113218716543815, -0.007887997236222589,
0.011445414468283652, -0.012028015650349323];
w = w_real + 1j*w_imag;

% Beam and null directions
theta_main = 0; % degrees
phi_main = 0; % degrees
theta_nulls = [15.0, 14.9, 30.0, 30.0, 45.0, 45.0, 60.0, 60.0]; % degrees
phi_nulls = [45.0, -135.0, 90.1, -89.9, 22.5, -157.5, 0.0, 180.0]; % degrees

% ================================================================
% STEP 1: COMPUTE E-PLANE PATTERN (for plotting and 1D metrics)
% ================================================================
theta_scan = linspace(-90, 90, 3601); % 0.05 deg resolution
phi_scan = phi_main * ones(size(theta_scan));
AF = zeros(size(theta_scan));
for i = 1:length(theta_scan)
th = theta_scan(i) * pi/180;
ph = phi_scan(i) * pi/180;
u_vec = [sin(th)*cos(ph); sin(th)*sin(ph); cos(th)];
sv = exp(1j * 2*pi * positions * u_vec);
AF(i) = w * sv;
end
AF_mag = abs(AF);
AF_peak = max(AF_mag);
AF_dB = 20*log10(AF_mag / AF_peak);

% ================================================================
% STEP 2: INDEPENDENTLY COMPUTE SLL (2D grid search)
% For planar/circular arrays, sidelobes can appear at any phi.
% A single E-plane cut would miss them. Full 2D search required.
% ================================================================
th_sll = linspace(-90, 90, 361) * pi/180;
ph_sll = linspace(-180, 180, 361) * pi/180;
P_sll = zeros(length(th_sll), length(ph_sll));
for i = 1:length(th_sll)
for j = 1:length(ph_sll)
u_vec = [sin(th_sll(i))*cos(ph_sll(j)); sin(th_sll(i))*sin(ph_sll(j)); cos(th_sll(i))];
sv = exp(1j * 2*pi * positions * u_vec);
P_sll(i,j) = abs(w * sv)^2;
end
end
% Find global peak
[P_global_max, glob_idx] = max(P_sll(:));
[gi, gj] = ind2sub(size(P_sll), glob_idx);
AF_peak_global = sqrt(P_global_max); % true global peak for normalization

% Mainbeam exclusion: angular distance from requested direction
```

```
% Formula matches engine: max(5 deg, rad2deg(1.5/sqrt(N))) * 2
th_main_rad = theta_main * pi/180;
ph_main_rad = phi_main * pi/180;
bw_est = max(5, 1.5/sqrt(N) * 180/pi); % matches engine formula (degrees)
P_sl = P_sll;
for i = 1:length(th_sll)
for j = 1:length(ph_sll)
cos_d = sin(th_sll(i))*sin(th_main_rad)*cos(ph_sll(j)-ph_main_rad) ...
+ cos(th_sll(i))*cos(th_main_rad);
ang_dist = acos(min(max(cos_d,-1),1)) * 180/pi;
if ang_dist < 2*bw_est
P_sl(i,j) = 0; % exclude mainbeam region
end
end
end
[peak_sl, sl_idx] = max(P_sl(:));
matlab_sll = 10*log10(peak_sl / P_global_max);

% ==========
=====================================================
% STEP 6: VERIFY MAINBEAM POINTING (local 2D peak search)
% For planar arrays with isotropic elements, the global peak may be a
% conjugate beam (back-lobe ambiguity). Search locally near the
% commanded direction -- this is what the engineer cares about.
% ================================================================
search_radius = 15; % degrees
th_local = linspace(max(-90, theta_main - search_radius), ...
min(90, theta_main + search_radius), 61) * pi/180;
ph_local = linspace(phi_main - search_radius, ...
phi_main + search_radius, 61) * pi/180;
P_local = zeros(length(th_local), length(ph_local));
for i = 1:length(th_local)
for j = 1:length(ph_local)
u_vec = [sin(th_local(i))*cos(ph_local(j)); sin(th_local(i))*sin(ph_local(j)); cos(th_local(i))];
sv = exp(1j * 2*pi * positions * u_vec);
P_local(i,j) = abs(w * sv)^2;
end
end
[~, loc_idx] = max(P_local(:));
[li, lj] = ind2sub(size(P_local), loc_idx);
% Refine: 51x51 around local peak for sub-degree accuracy
dth_r = (th_local(2)-th_local(1)) * 1.5;
dph_r = (ph_local(2)-ph_local(1)) * 1.5;
th_ref = linspace(max(-pi/2, th_local(li)-dth_r), min(pi/2, th_local(li)+dth_r), 51);
ph_ref = linspace(ph_local(lj)-dph_r, ph_local(lj)+dph_r, 51);
P_ref = zeros(length(th_ref), length(ph_ref));
for i = 1:length(th_ref)
for j = 1:length(ph_ref)
u_vec = [sin(th_ref(i))*cos(ph_ref(j)); sin(th_ref(i))*sin(ph_ref(j)); cos(th_ref(i))];
sv = exp(1j * 2*pi * positions * u_vec);
P_ref(i,j) = abs(w * sv)^2;
end
end
[~, ref_idx] = max(P_ref(:));
[ri, rj] = ind2sub(size(P_ref), ref_idx);
actual_peak_theta = th_ref(ri); % radians -- used by beamwidth step
actual_peak_phi = ph_ref(rj); % radians -- used by beamwidth step
matlab_peak_theta = actual_peak_theta * 180/pi;
matlab_peak_phi = actual_peak_phi * 180/pi;
% Angular distance in 3D (great circle) from commanded to local peak
cos_pt = sin(actual_peak_theta)*sin(th_main_rad)*cos(actual_peak_phi-ph_main_rad) ...
+ cos(actual_peak_theta)*cos(th_main_rad);
matlab_pointing_err = acos(min(max(cos_pt,-1),1)) * 180/pi;

% Update AF_peak to global peak for null depth normalization and E-plane plot
AF_peak = AF_peak_global;
AF_dB = 20*log10(AF_mag / AF_peak); % recompute E-plane dB with true global peak

% ================================================================
% STEP 3: INDEPENDENTLY COMPUTE DIRECTIVITY
% D = 4*pi * |AF(main)|^2 / integral(|AF|^2 sin(theta) dtheta dphi)
% Full sphere integration -- the universal definition of directivity.
% ================================================================
theta_int = linspace(1e-4, pi-1e-4, 181); % avoid poles
phi_int = linspace(0, 2*pi, 361);
dth = theta_int(2) - theta_int(1);
```

```
dph = phi_int(2) - phi_int(1);
total_power = 0;
for i = 1:length(theta_int)
th = theta_int(i);
for j = 1:length(phi_int)
ph = phi_int(j);
u_vec = [sin(th)
*cos(ph); sin(th)*sin(ph); cos(th)];
sv = exp(1j * 2*pi * positions * u_vec);
af_val = abs(w * sv)^2;
total_power = total_power + af_val * sin(th) * dth * dph;
end
end
% Mainbeam power
th_m = theta_main*pi/180; ph_m = phi_main*pi/180;
u_main = [sin(th_m)*cos(ph_m); sin(th_m)*sin(ph_m); cos(th_m)];
sv_main = exp(1j * 2*pi * positions * u_main);
P_main = abs(w * sv_main)^2;
matlab_dir_dBi = 10*log10(4*pi * P_main / total_power);

% ================================================================
% STEP 4: INDEPENDENTLY COMPUTE BEAMWIDTH (-3 dB points)
% IEEE Std 145-2013: cut through the ACTUAL peak, not commanded.
% Normalized to GLOBAL 2D peak. Uses actual_peak_theta/phi from Step 6.
% ================================================================
% Theta cut: vary theta at fixed phi = actual peak phi
bw_theta_scan = linspace(-90, 90, 7201);
bw_AF = zeros(size(bw_theta_scan));
for i = 1:length(bw_theta_scan)
th = bw_theta_scan(i) * pi/180;
ph = actual_peak_phi; % cut through actual peak
u_vec = [sin(th)*cos(ph); sin(th)*sin(ph); cos(th)];
sv = exp(1j * 2*pi * positions * u_vec);
bw_AF(i) = abs(w * sv);
end
bw_AF_dB = 20*log10(bw_AF / AF_peak_global); % normalize to GLOBAL peak
[~, bw_main_idx] = max(bw_AF);
bw_left = NaN; bw_right = NaN;
for i = bw_main_idx:-1:2
if bw_AF_dB(i) <= -3
frac = (-3 - bw_AF_dB(i)) / (bw_AF_dB(i+1) - bw_AF_dB(i));
bw_left = bw_theta_scan(i) + frac*(bw_theta_scan(i+1) - bw_theta_scan(i));
break;
end
end
for i = bw_main_idx:length(bw_AF_dB)-1
if bw_AF_dB(i) <= -3
frac = (-3 - bw_AF_dB(i)) / (bw_AF_dB(i-1) - bw_AF_dB(i));
bw_right = bw_theta_scan(i) + frac*(bw_theta_scan(i-1) - bw_theta_scan(i));
break;
end
end
matlab_bw = abs(bw_right - bw_left);

% ================================================================
% STEP 5: VERIFY NULL DEPTHS
% ================================================================
fprintf('\n');
fprintf('================================================================\n');
fprintf(' INDEPENDENT VERIFICATION -- MATLAB R%s\n', version('-release'));
fprintf(' URA %d-element Array, Mainbeam %g deg\n', N, theta_main);
fprintf('================================================================\n');

fprintf('\n--- NULL DEPTH VERIFICATION ---\n');
null_depths_matlab = zeros(1, length(theta_nulls));
null_all_pass = true;
for k = 1:length(theta_nulls)
th = theta_nulls(k) * pi/180;
ph = phi_nulls(k) * pi/180;
u_vec = [sin(th)*cos(ph); sin(th)*sin(ph); cos(th)];
sv = exp(1j * 2*pi * positions * u_vec);
af_null = w * sv;
depth_dB = 20*log10(abs(af_null) / AF_peak);
null_depths_matlab(k) = depth_dB;
pass_fail = 'PASS';
if depth_dB > -30; pass_fail = '** FAIL **'; null_all_pass = false; end
```

```
fprintf(' Null at (%+6.1f, %+5.1f) deg: %8.1f dB [%s]\n', ...
theta_nulls(k), phi_nulls(k), depth_dB, pass_fail);
end
matlab_worst_null = max(null_depths_matl
ab);

% ================================================================
% VERIFICATION SUMMARY -- PASS/FAIL TABLE
% Every value below was computed independently by MATLAB.
% Nothing is taken from the report on trust.
% ================================================================
fprintf('\n');
fprintf('================================================================\n');
fprintf(' VERIFICATION SUMMARY\n');
fprintf('================================================================\n');
fprintf(' %-22s %10s %10s %8s\n', 'METRIC', 'CLAIMED', 'MATLAB', 'STATUS');
fprintf(' %-22s %10s %10s %8s\n', '------', '-------', '------', '------');

% SLL -- tolerance 0.5 dB (accounts for scan resolution)
sll_match = abs(matlab_sll - claimed_sll_dB) < 0.5;
fprintf(' %-22s %9.1f %9.1f %8s\n', 'SLL (dB)', ...
claimed_sll_dB, matlab_sll, tf_str(sll_match));

% Directivity -- tolerance 0.3 dBi (numerical integration resolution)
dir_match = abs(matlab_dir_dBi - claimed_dir_dBi) < 0.3;
fprintf(' %-22s %9.1f %9.1f %8s\n', 'Directivity (dBi)', ...
claimed_dir_dBi, matlab_dir_dBi, tf_str(dir_match));

% Beamwidth -- tolerance 0.5 deg
bw_match = abs(matlab_bw - claimed_bw_deg) < 0.5;
fprintf(' %-22s %9.1f %9.1f %8s\n', 'Beamwidth (deg)', ...
claimed_bw_deg, matlab_bw, tf_str(bw_match));

% Pointing error -- tolerance 0.1 deg
pt_match = abs(matlab_pointing_err - claimed_pointing_err) < 0.1;
fprintf(' %-22s %9.3f %9.3f %8s\n', 'Pointing Error (deg)', ...
claimed_pointing_err, matlab_pointing_err, tf_str(pt_match));

% Null depths -- both must be below -30 dB
fprintf(' %-22s %9.1f %9.1f %8s\n', 'Worst Null (dB)', ...
claimed_worst_null, matlab_worst_null, tf_str(null_all_pass));

fprintf(' %-22s %10s %10s %8s\n', '------', '-------', '------', '------');
all_pass = sll_match && dir_match && bw_match && pt_match && null_all_pass;
if all_pass
fprintf('\n >>> ALL METRICS VERIFIED -- CLAIMS CONFIRMED <<<\n');
else
fprintf('\n >>> VERIFICATION FAILED -- SEE ABOVE <<<\n');
end

% -- Note on null depth precision --
fprintf('\n--- NOTE ON NULL DEPTH VALUES ---\n');
fprintf(' Claimed: %.1f dB | MATLAB: %.1f dB | Delta: %.1f dB\n', ...
claimed_worst_null, matlab_worst_null, ...
abs(matlab_worst_null - claimed_worst_null));
fprintf(' Both values are far below the -30 dB requirement.\n');
fprintf(' Differences below -280 dB are expected: at these depths\n');
fprintf(' we are at the float64 machine precision floor (~1e-16).\n');
fprintf(' Different evaluation paths (matrix multiply vs loop) produce\n');
fprintf(' different rounding at the 15th-16th significant digit.\n');
fprintf(' This has zero operational impact -- real hardware limits\n');
fprintf(' null depth to approximately -40 to -60 dB.\n');

fprintf('\n================================================================\n');
fprintf(' VERIFICATION COMPLETE\n');
fprintf('================================================================\n');

% Helpe
r function
function s = tf_str(v)
if v; s = 'PASS'; else; s = '** FAIL **'; end
end

% ================================================================
% PLOT: E-plane pattern with SLL annotation
% ================================================================
```

```
figure('Name', 'E-plane Pattern -- Independent Verification', 'Color', 'w');
plot(theta_scan, AF_dB, 'b-', 'LineWidth', 2); hold on;
yline(matlab_sll, 'm--', sprintf('SLL = %.1f dB (MATLAB)', matlab_sll), ...
'LineWidth', 1.5, 'LabelHorizontalAlignment', 'left');
yline(-3, 'k--', '-3 dB (beamwidth)');
yline(-30, 'r:', '-30 dB (null threshold)', 'LineWidth', 1);
for k = 1:length(theta_nulls)
xline(theta_nulls(k), 'r--', sprintf('null %+.0f deg', theta_nulls(k)), ...
'LineWidth', 1.5);
end
xline(theta_main, 'g-.', 'mainbeam', 'LineWidth', 1.5);
xlabel('\theta (degrees)'); ylabel('Pattern (dB)');
title(sprintf('URA %d-element -- MATLAB Verified: SLL=%.1f dB, Dir=%.1f dBi, BW=%.1f deg', ...
N, matlab_sll, matlab_dir_dBi, matlab_bw));
grid on; ylim([-60 5]);
legend('Pattern', 'Location', 'best');

% -- 2D Pattern Heatmap (theta x phi) -- planar array --
theta_2d = linspace(-60, 60, 121);
phi_2d = linspace(-90, 90, 91);
P_2d = zeros(length(theta_2d), length(phi_2d));
for i = 1:length(theta_2d)
for j = 1:length(phi_2d)
th = theta_2d(i)*pi/180; ph = phi_2d(j)*pi/180;
u_vec = [sin(th)*cos(ph); sin(th)*sin(ph); cos(th)];
sv = exp(1j*2*pi*positions*u_vec);
P_2d(i,j) = abs(w * sv)^2;
end
end
P_2d_dB = 10*log10(P_2d / max(P_2d(:)));
figure('Name', '2D Pattern -- Independent Verification', 'Color', 'w');
imagesc(phi_2d, theta_2d, P_2d_dB); colorbar;
caxis([-40 0]); xlabel('\phi (deg)'); ylabel('\theta (deg)');
title(sprintf('URA 2D Pattern -- Verified (dB)'));
hold on;
plot(phi_main, theta_main, 'g*', 'MarkerSize', 15, 'LineWidth', 2);
for k = 1:length(theta_nulls)
plot(phi_nulls(k), theta_nulls(k), 'rx', 'MarkerSize', 12, 'LineWidth', 2);
end
```


Verify: AF(θ) = Σ w_k · exp(j·k·d_norm·sin(θ))